A.15 Constrained Optimization & APPDX. FOR Ch. 5

Some economics problems involve finding a maximum or a minimum subject to a constraint:

For instance:

Maximize utility subject to your budget constraint.

Example:

Sonya's preferences over cans of beer (b) and cans of cola (c) are given by the following utility function:

$u(b,c)=bc$

She has \$100 to spend. Cola costs \$1 per can. Beer costs \$2 per can. (Note: These assumptions imply that her budget line is given by the equation $c+2b=100$).

What is the best bundle of beer and cola Sonya can choose?

This question can be restated as the following constrained optimization problem:

max $u(b,c)=bc$

such that $c+2b=100$

The "objective function" in this problem is: $u(b,c)=bc$

The "constraint" in this problem is: $c+2b=100$

Next: An illustration of one method for solving a constrained optimization problem (in particular, the "substitution method").

Consider the constrained optimization problem for Sonya:

Max $u(b,c)=bc$

such that $c+2b=100$

Using the substitution method:

Note that we can rewrite the budget constraint as $c = 100 - 2b$.

Substitute this rewritten version of the budget constraint into the *objective* function. We then have the following optimization problem (where there is just one variable):

Max $u(b) = b(100 - 2b) = 100b - 2b^2$

Find the first-order condition:

$\frac{du}{db}=100-4b$; Set this equal to zero.

We obtain $b=25$.

Next, check the second-order condition:

At $b=25$, the second derivative of u is -4. So the second-order condition is satisfied.

Substitute $b=25$ back into the budget constraint.

We obtain $c=50$.

The optimal bundle for Sonya is 25 cans of beer and 50 cans of cola.